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Lindberg, Erik

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Sinusoidal Excitation on the Chua's Circuit Simulation of Limit Cycles and Chaos

Erik Lindberg
Inst. of Circuit Theory and Telecommunication
343 Technical University of Denmark, DK-2800 Lyngby, Denmark

e-mail: TTTEL at VM.UNI-C.DK
fax: +45 4593 0355, telephone: +45 4593 1222 - 3650 direct

1. SUMMARY

Experiments with modelling and simulation of sinusoidal excitation on the Chua's Circuit are presented. It is demonstrated that the behaviour of the circuit is based on the interaction of two different kinds of energy balance: (1) Chaotic behaviour based on a balance between two unstable "states of charging" and (2) Stable limit cycle behaviour based on the balance between the energy lost in the regions with mainly positive losses and the energy gained in the regions with mainly negative losses. Convergence problems observed in connection with simulation of the ideal piecewise-linear model are solved by means of a smooth continuous model of the non-linear element based on the ideal operational amplifier model instead of a polynomial approximation. The movements of the eigenvalues as functions of the nonlinear resistance are found. Stable limit cycles of periods 1, 2, 3 and 5 are found by introducing losses.

2. INTRODUCTION

In April 1992 K. Murali and M. Lakshmanan published a paper on a hardware experiment describing the effect of sinusoidal excitation on the Chua's Circuit [ref. 1]. The aim of this contribution is to verify by means of simulation some of the findings of K. Murali and M. Lakshmanan and present a simple physical explanation of the behaviour of the circuit.

With reference to Fig.1 the circuit may be described as an ideal harmonic oscillator (L2 in parallel with C2) which is driven by a voltage source: $F \cdot \sin(2\pi \cdot f \cdot time)$ in series with a coil L1. The coil L1 is much greater than L2. The harmonic oscillator is loaded by a frequency dependent load made from a resistor R in series with a parallel RC-circuit (C1 in parallel with RNL) where RNL (The Chua Diode) is an almost piecewise-linear negative resistance realized by means of an operational amplifier, a pair of diodes, seven resistors and DC supply voltages of +9V and -9V [1]. The resonance frequencies of the harmonic oscillator is 1248.514093 Hz excluding L1 and 1346.139070 Hz including L1. In Fig.1 RNL is modelled as a negative dynamic resistor Rd in parallel with a Thevenin current source ITh.

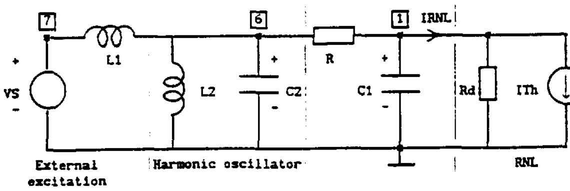


Fig.1, Chua's Circuit with external excitation. L1=80mH, L2=13mH, C2=1.25μF, R=1310 Ω, C1=0.017μF

3. THE NONLINEAR RESISTOR

For small values of the voltage of C1 the characteristic of RNL is a 3-region piecewise-linear characteristic Fig.2(a). The positions of the breakpoints of the 3-region characteristic is very sensitive to variation in the saturation current Is of the two diodes. Under the assumption that the operational amplifier is ideal by means of simulation the nonlinear resistor is found to be approximately picewise-linear as follows:

REGION 1: $IR > +0.94$ mA, $VR < -1.13$ V

$$IR = \alpha * VR + \beta \text{ where } 1/\alpha = -1954\ \Omega \text{ and } \beta = +0.36 \text{ mA}$$

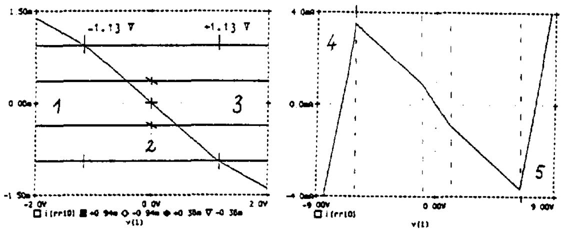
REGION 2:
$+0.94\text{mA} > IR > -0.94\text{mA}$, $-1.13\text{ V} < VR < +1.13\text{ V}$

$$IR = \alpha * VR + \beta \text{ where } 1/\alpha = -1200\ \Omega \text{ and } \beta = 0 \text{ mA}$$

REGION 3: $IR < -0.94$ mA, $VR > +1.13$ V

$$IR = \alpha * VR + \beta \text{ where } 1/\alpha = -1954\ \Omega \text{ and } \beta = -0.36 \text{ mA}$$

In order to obtain a continuous movement of the eigenvalues and in order to avoid mixing of phenomenas due to the ideal 3-region almost piecewise-linear charateristic with phenomenas due to the internal parameters of the operational amplifier an ideal operational amplifier model is used in the simulations and the regions with positive slope is modelled by means of a simple voltage controlled current source placed in parallel to C1.



(a) 3-region (b) 5-region

Fig.2, Characteristic of the non-linear resistor RNL.
x-axis: voltage VR, y-axis: current IR

Fig.2(b) shows the 5-region almost piecewise-linear characteristic of RNL when the operational amplifier model is based on Ebers-Moll models for the internal transistors.

4. THE EIGENVALUES

Fig.3 shows the movement of the eigenvalues of the circuit as functions of the nonlinear resistance RNL. During the transitions between regions 1 and 3 (RNL = $-1954\ \Omega$) and region 2 (RNL = $-1200\ \Omega$) all the eigenvalues are in the left half plane when $-1340.43\ \Omega <$ RNL $< -1310\ \Omega$. Note that the value $1310\ \Omega$ is the value of the resistor R in Fig.1.

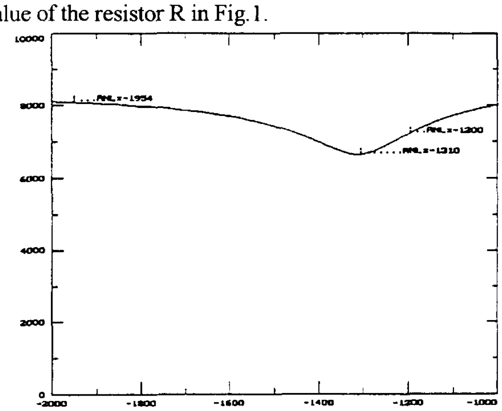


Fig.3(a), Imaginary part of complex pole pair as function of RNL resistor Rd

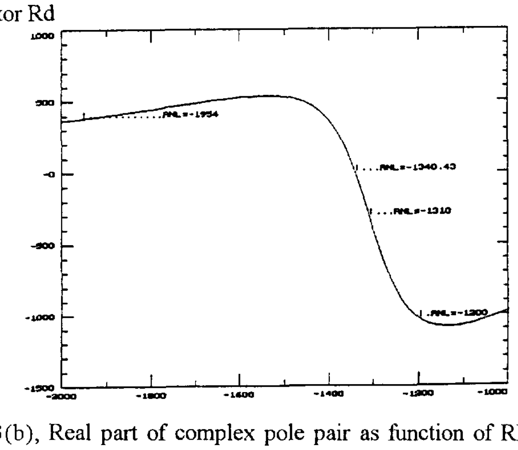


Fig.3(b), Real part of complex pole pair as function of RNL resistor Rd

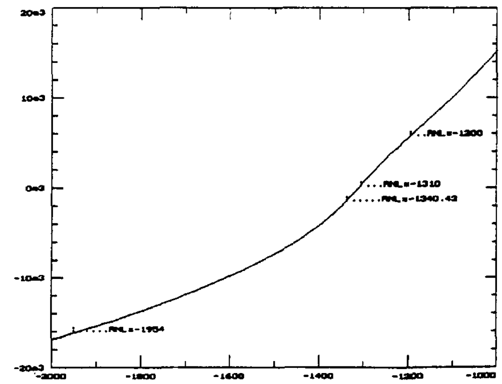


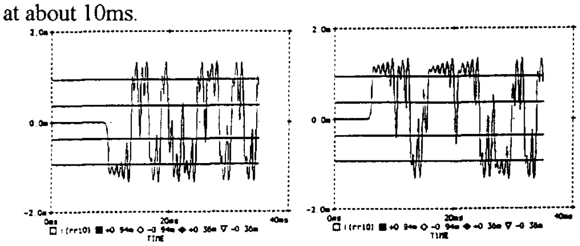
Fig.3(c), Real pole as function of RNL resistor Rd

5. A CIRCUIT THEORETIC DISCUSSION

As expected [3] in regions 1 and 3 we have a complex pair of poles in the right half plane (RHP) and a simple pole in the left half plane (LHP). In region 2 we have a simple pole in RHP and a complex pair of poles in LHP. The autonomous piecewise-linear circuit is unstable because of the poles in RHP. If we start up in origo (region 2) an exponential rise of the voltage across C1 is to be expected due to the simple pole in RHP. When region 1 is entered the complex pair of poles switches to RHP and an exponentially rising sine oscillation of about 1286 Hz is to be expected [2].

When the amplitude of the oscillations grows we enter region 2 again and we observe a growing nonlinear oscillation around a DC level of about -1.13 V corresponding to the breakpoint of the piecewise-linear characteristic. Energy is supplied from the Thevenin source (β = +0.36 mA). When sufficient energy is supplied from the Thevenin source the posibility of switching to region 3 occur. Here the Thevenin source changes its direction (β = -0.36 mA) and the charging of the capacitor C1 goes in the opposite direction with oscillations around a DC level of +1.13 V corresponding to the other breakpoint.

As a conclusion we might expect the posibility of stable oscillations with energy balance between the two unstable "DC levels of oscillations". The simulation of this behavior is presented in Fig.4(a) (ideal op. amp.) and Fig.4(b) (piecewise-linear voltage controlled current source). The piecewise-linear model enters region 3 at about 6ms where the continous model enters region 1 at about 10ms.



(a) continous model (b) piecewise-linear model

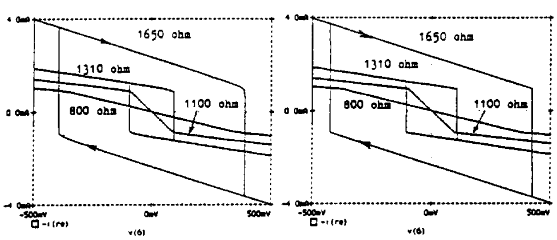
Fig.4, Current i(rr10) = IRNL as funtions of time

6. NUMERICAL COMMENTS

Problems with no convergence were observed in connection with simulation of the ideal piecewise-linear model (immediate switch of the eigenvalues may give rise to a jump in the voltage across C1 ?). The number of integration steps in Fig.4 is 1500 for the continuous model and 1748 for the piecewise-linear model. - As expected the piecewise-linear model uses a 2'nd order method the most. The integration method used is a modified Gear method. The relative tolerance of the integration is 1e-6. In the simulations presented in Fig.4(a) and Fig.4(b) the maximum integration step is 1ms and the initial integration step is 35e-9 seconds. If the maximum integration step is 35ms and the initial integration step is 50e-9 seconds the results become very different due to the sensitivity of the solution with respect to initial conditions. The piecewise-linear model Fig.4(b) is much more sensitive than the continuous model Fig.4(a). The minimum integration step allowed is 1e-20 seconds.

7. THE NONLINEAR LOAD OF THE HARMONIC OSCILLATOR

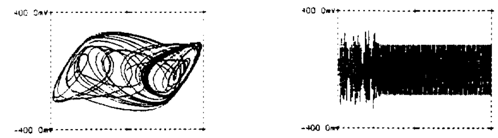
Fig.5(a) shows the nonlinear load characteristic of the harmonic oscillator when the resistor R is varied assuming the values 800, 1100, 1310 and 1650 Ω. The ideal op.amp. model is used. Fig.5(b) is the same as Fig.5(a) but with use of the piecewise-linear model. Note that the hysteresis loops become slightly larger in Fig.5(b) than in Fig.5(a). If you zoom around origo you will find that the apparently continuous curves for R=1100 Ω and R=800 Ω also are hysteresis loops due to the still existing "switching" Thevenin source.



(a) continuous model (b) piecewise-linear model
Fig.5, Nonlinear load, x-axis: v(6) = VC2, y-axis: -i(re) = IR

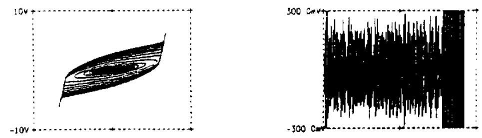
8. SEARCH FOR LIMIT CYCLES

A large number of simulations have been made both without and with a series resistance RL2 in connection with the coil L2 of the harmonic oscillator. With RL2 = 0 Ω only period-1 limit cycles have been found. Two small ones corresponding to oscillation around one of the breakpoints and a large one corresponding to oscillations between the regions with positive slope of RNL. Fig.6(a) shows the voltage VC2 as a function of the voltage VC1 and Fig.6(b) shows VC2 as a function of time when a voltage VS with F = 200mV and f = 1286 Hz is applied.

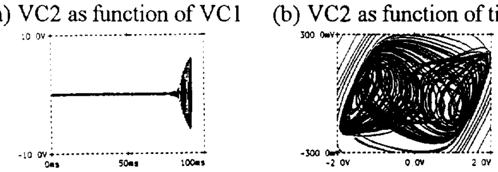


(a) VC2 as function of VC1 (b) VC2 as function of time
Fig.6, external amplitude F=200mV

The sensitivity is enormous, e.g. F = 227.789500mV gives one and F = 227.789506mV gives the other of the two different small limit cycles after about 60ms of chaos. Fig.7 shows the same as Fig.6 but for F = 228mV. It takes about 80ms of chaos (Fig.7(c)) before transition to the large limit cycle occur. Fig.7(d) shows a zoom of Fig.7(a).



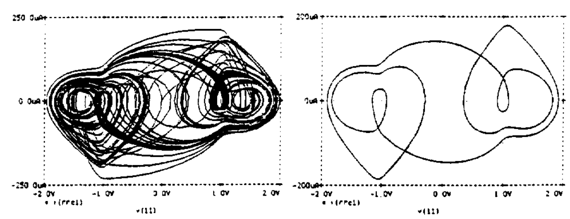
(a) VC2 as function of VC1 (b) VC2 as function of time



(c) VC2 as function of time (d) zoom of Fig.7(a)
Fig.7, external amplitude F=228mV

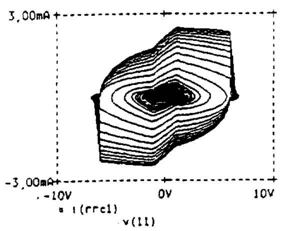
Without a loss resistance RL2 it is apparently impossible to obtain the bifurcation diagram in the F-f plane found by K. Murali and M. Lakshmanan shown in Fig.3 of their paper [1]. The boundary between the area with the two small limit cycles and the area with the large limit cycle searched for in [2] seems not to be a line in the sense of Euclid but a narrow band - a "coast line" - where it is impossible to distinguish between chaos due to the accuracy of the calculations and chaos due to the nature of the system. Unfortunately K. Murali and M. Lakshmanan give no information about the loss resistors of L2 and L1.

By chance a resistor RL2 = 1 Ω was introduced as a first attempt to study the influence of losses. For some reason a proper limit cycle (derivative of signal as function of signal, IC1 as function of VC1) was asked for. Fig.8 shows the result, a beautiful period-5 **"twin heart"** limit cycle.

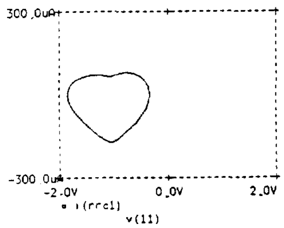


(a) transition to limit cycle (b) period-5 limit cycle
Fig.8, The current of the capacitor C1 as function of the voltage of C1

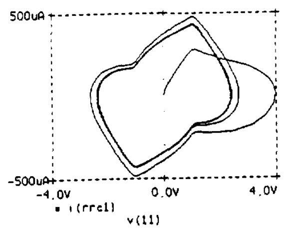
The amplitude of the external signal is F=150mV, the angular frequency is $2\cdot\pi\cdot f = 8.0822898994674e{+}3$ rps corresponding to the imaginary part of the complex pair of poles in regions 1 and 3 [2]. The simulation limit used is 15000 integration steps. Chaos is observed between 0ms and 40ms and the period-5 limit cycle is observed between 40ms and 268.8ms. In the physical realisation of the circuit it is necessary to introduce a resistance of 1 Ω in series with C1 for measurement of the current of C1. The result of the simulation in this case is chaos between 0ms and 235ms and a period-5 limit cycle with starting bifurcation between 235ms and 293.5ms. If the angular frequency is changed to 8.0822800000000e+3 rps chaos is observed between 0ms and 90ms and a period-5 limit cycle without starting bifurcation between 90ms and 274.9ms with 15000 integration steps as simulation limit.



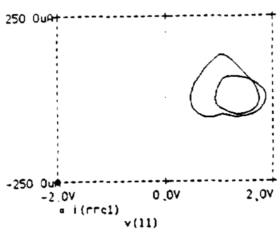
(a) large period-1 limit cycle



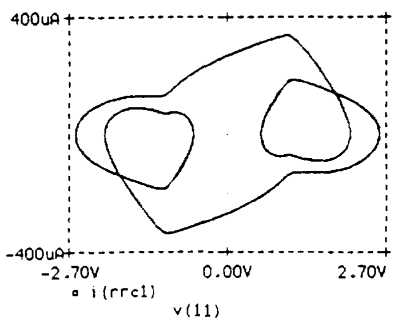
(b) small period-1 limit cycle



(c) small period-1 limit cycle



(d) period-2 limit cycle



(e) period-3 limit cycle

Fig.9, The current IC1 of capacitor C1, i(rrc1), as function of the voltage VC1 of capacitor C1, v(11)

Fig.9 shows examples of period-1, period-2 and period-3 limit cycles. With reference to Fig.7(a) of K. Murali and M. Lakshmanan [1] a period-4 limit cycle should be found for an excitation of F = 516.9e-3 volts at the frequency f = 820 Hz. RL2 is chosen as 80 Ω and the result in this case is a period-3 limit cycle, Fig.10.

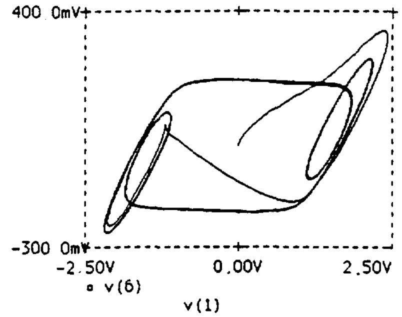


Fig.10, VC2 as function of VC1,
RL2 = 80 Ω, RRC1 = 0 Ω, F = 516.19mV, f = 820 Hz

9. Conclusion

Simulation of the Murali-Lakshmanan experiment with sinusoidal excitation of the Chua's Circuit has been performed. The behaviour of the Chua's Circuit is explained as a very sensitive balance of energy related to two unstable "states of capacitor charging". With lossless coils the border between small and large period-1 limit cycles appears to be a "coast line" where it is impossible to distinguish between chaos due to the accuracy of the calculations and chaos due to the nature of the system. When losses are introduced in the coils this "coast line" broadens into a "bifurcation sea" with islands of high period limit cycles verifying the bifurcation diagram of K. Murali and M. Lakshmanan. Search for period-4 limit cycles is going on.

10. References

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